Announcement of the lecture

Higher Analysis (4+2 hrs/week, 10 ECTS)

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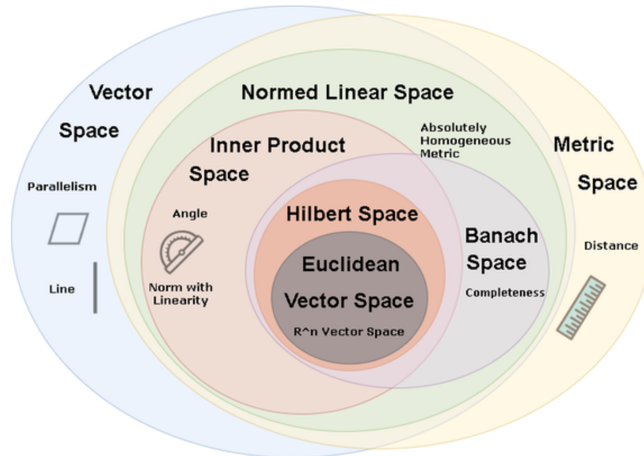
Winter term 2025/26

Contents:

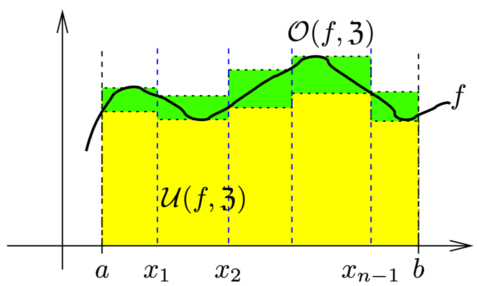
- Metric, normed, and Banach spaces
- Basic topological concepts: open/closed sets, continuity, compactness
- Examples: Sequence spaces, function spaces
- Theorem of Stone-Weierstraß, Banach's Fixed Point Theorem
- Lebesgue measure and integration, convergence theorems
- Lebesgue spaces L_p
- Bounded linear operators: Functionals, dual spaces, Hahn-Banach Theorem
- Hilbert spaces: Definitions, Orthonormal bases, Fourier transformation, Sobolev spaces
- Spectral theory of compact operators

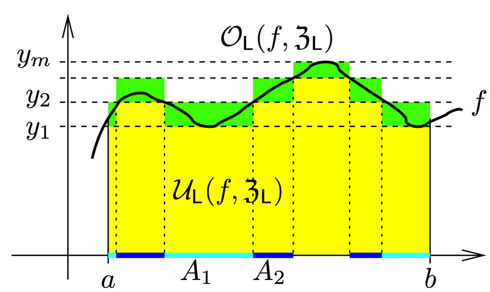


Description:

This lecture attempts to provide an insight into analysis beyond the lectures of the first two semesters. It attempts to give an introduction to modern functional analysis as well as measure and integration theory. It covers the basic concepts and results of these areas, which are potentially relevant (or even indispensable) for all students who wish to attend further advanced math courses, e.g. the lecture *'Neural network theory'* in Summer term 2026 (and many others).

Starting with metric, normed, and Banach spaces, we study convergence, continuity, and compactness. Classical function and sequence spaces illustrate these abstract notions. Moreover, the course introduces Lebesgue measure and integration, laying the groundwork for a rigorous understanding of L_p spaces and the associated convergence theorems.





We further explore the theory of bounded linear operators, including the dual space as well as the Hahn-Banach Theorem. An introduction to Hilbert spaces covers orthonormal bases, Fourier transformation, and Sobolev spaces, with applications to spectral problems and compact operators.

Prerequisites: Linear Algebra I and Analysis I+II or Mathematics for Data Science I+II or comparable are recommended. This course is suitable for Bachelor students in Mathematics/Data Science as well as for the Master Data Science/CAM.

Creditable as: Theoretical/Applied Mathematics (for B.Sc. Mathematics); Specialisation Mathematical Theory/Foundations of Data Science (for Data Science); Specialisation MApA (Modeling and applied analysis)/ NASi (Numerical analysis and simulation)/ Non-specialisation modules/ Free elective modules (for CAM)

Preparation for: (Numerics of) Partial differential equations I+II; Modeling in analysis and continuum mechanics I+II; Navier Stokes equations; Neural network theory

Literature

[Rud91] W. Rudin. *Functional Analysis* McGraw-Hill, 1991.

[Rud70] W. Rudin. *Real and complex analysis* McGraw-Hill, 1970.

[Con07] J.B. Conway. *A course in functional analysis.* Springer, 2007.

[SS05] E.M. Stein, R. Shakarchi. *Real analysis: Measure theory, integration, and Hilbert spaces.* Princeton Lectures, 2005.

[Wer09] D. Werner. *Höhere Analysis.* Springer, 2009 (German).